# MATH-382 Differential Geometry

**Credit Hours:** 3-0

**Prerequisite:** None

**Course Objectives:** After having completed this course, the students would be expected to understand classical concepts in the local theory of curves and surfaces. Also the students will befamiliar with the geometrical interpretation of the terminology used in the course.

**Core Contents:** Parametric representation of curves and surfaces, tangent and normal vectors, curvatures, fundamental forms.

**Detailed Course Contents**: Nature and purpose of differential geometry, concept of mapping. Coordinates in Euclidean space, vectors in Euclidean space, basic rules of vector calculus in Euclidean space, concept of a curve in differential geometry, examples of special curves. Arc length, tangent and normal plane, osculating plane, principal normal, curvature, osculating circle. Binormal, moving trihedron of a curve, torsion. Formulae of Frenet, motion of the trihedron, vector of Darboux.

Spherical images of a curve, contact, osculating sphere, natural equations of a curve, examples of curves and their natural equations. Involutes and evolutes, Bertrand curves. Concept of a surface in differential geometry curves on a surface, tangent plane to a surface. First fundamental form. Concept of Riemannian geometry. Summation convention, properties of the first fundamental form. Contravariant and covariant vectors, Contravariant, covariant, and mixed tensors (Concepts of tensor from theses sections). Normal to a surface, definition of normal section and curvature with some results. Second fundamental form, Arbitrary and normal sections of a surface. Meusnier's theorem. Asymptotic lines, are elliptic, parabolic, and hyperbolic points of a surface. Principal curvature, lines of curvature, Gaussian and mean curvature. Formulae of Weingarten and Gauss. Fundamental theorem of theory of surfaces.

**Course Outcomes:**

- Student should be able to understand the concept of a curve in differential geometry.
- Student should know the Frenet-Serret theorem and their applications.
- Student should be familiar with concept of surfaces in differential geometry.
- Student should be able to understand fundamental forms.

**Text Book:** Andrew Pressley (2ed) Elementary Differential Geometry-Springer (2010).

**Reference Books:**


1. Erwin Kreyszig, Differential Geometry, Dover Publications, Inc. New York, (1959).
2. R.S. Millman, G. D. Parker, Elements of Differential Geometry, Prentice-Hall Inc, (1977).
3. Alfred Gray, Modern Differential Geometry of Curves and Surfaces,

Chapman &Hall/CRC (2005).

| Weekly Breakdown | | |
|---|---|---|
| ***Week*** | ***Section*** | ***Topics*** |
| 1 | 1.1, 1.2 | Parametrization of curves in R2, Tangent vector, Arc-Length, Unit speed curves. |
| 2 | 1.3, 1.4, | Reparametrization of curves, Singular points, Regular curves, Closed curves. |
| 3 | 2.1 | Curvature of curves, Curvature of space curves, derivation of formula for curvature. |
| 4 | 2.2 | Signed unit normal, Turning angle of curve, Signed curvature. Osculating circle, centre of curvature, radius of curvature, envelope of curves, involute and evolute of curve. |
| 5 | 2.3 | Space curves: torsion, principal normal, binormal, derivation of formula for torsion for non-unit speed curves, some particular cases, Derivation of Frenet-Serret equations, Generalized helix. |
| 6 | 3.1,3.2 | Global properties of curves: Simple closed curves, positively oriented curves. Wirtinger's inequality, The isoperimetric inequality. |
| 7 | 4.1, 4.2 | Introduction to surfaces, parametrization of surfaces: ellipsoid, paraboloid, hyperboloid, helicoid, torus, etc. Smooth surfaces, regular surfaces, allowable surfaces, reparameterization of surfaces. |
| 8 | 4.3-4.5 | Smooth maps, diffeomorphism and diffeomorphic surfaces. Tangents and derivatives, tangent plane, parametric curves on surfaces. Normals and orientability. |
| 9 | **Mid Semester Exam** | |
| 10 | 5.1,5.2 | Level surfaces, Quadric surfaces, Parametrization of quadric surfaces in nonstandard form. |
| 11 | 5.3, 6.1 | Ruled surfaces and surface of revolutions. Length of curves on surfaces, First fundamental form. |
| 12 | 6.2, 6.3 | Isometries of surfaces, tangent developable. Conformal mappings of surfaces, Surface area by first fundamental form, invariants of first fundamental forms. |
| 13 | 7.1, 7.2 | The second fundamental form, The Gauss and Weingarten maps. |
| 14 | 7.3,7.4 | The normal and geodesic curvatures, normal section, asymptotic curves on surfaces, Meusnier's theorem, parallel transport and covariant derivative. |
| 15 | 8.1, 8.2 | Gaussain and mean curvatures, minimal surface, Principal curvatures. Elliptic, parabolic and hyperbolic points, line of curvature, umbilic points, principal directions and principal directions. |
| 16 | 9.1-9.2 | Geodesics: definition and basic properties, Geodesic equations. |
| 17 | | Review |
| 18 | **End Semester Exam** | |